D-concurrence bounds for pair coherent states

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Abstract

The pair coherent state is a state of a two-mode radiation field which is known as a state with non-Gaussian wave function. In this paper, the upper and lower bounds for D-concurrence (a new entanglement measure) have been studied over this state and calculated.

Keywords: Pair coherent state; D-concurrence; Entanglement.

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1 Introduction

Quantum information processing has been the focus of recent quantum scientific research and has attracted a lot of attention. Quantum entanglement is one of the key resource for quantum information processing and manipulating of entangled states are essential for quantum information applications. Such these applications are quantum teleportation [1, 2], quantum cryptography [3, 4], quantum dense coding [5, 6, 7], and quantum computation [8, 9, 10]. The fundamental question in quantum entanglement theory is which states are entangled and which ones are not? Only in some cases we can find the simple answer to this question. The case of pure bipartite states is the simplest ones. Any bipartite pure state $|\Psi_{AB}\rangle \in H_{AB} = H_A \otimes H_B$ is called separable (entangled) iff it can be (can not be) written as a product of two vectors corresponding to Hilbert spaces of subsystems: $|\Psi_{AB}\rangle = |\phi_A\rangle|\psi_B\rangle$.

Bennett et *al* [11] has defined a measure of entanglement for each pure state of a bipartite system $|\rho_{AB}\rangle$ as below:

$$E(|\rho_{AB}\rangle) = -Tr(\rho_A \log_2 \rho_A) = -Tr(\rho_B \log_2 \rho_B), \tag{1-1}$$

which is called as entropy of entanglement. $\rho_A = Tr_B|\rho_{AB}\rangle\langle\rho_{AB}|$ is the partial trace of ρ over subsystem B, and ρ_B has a similar meaning. Some measures such as concurrence [12, 13, 14, 15], negativity [16, 17, 18], and tangle [19, 20, 21] can be used for quantifying entanglement. Experimental quantification of entanglement has attracted more attention recently [22, 23, 24, 25]. D-concurrence is a measures for quantifying the amount of entanglement which have proposed by Ma and Zhang [26]. This measure has advantages in comparison with other measures, specially concurrence, such as simplicity of form and accuracy of results.

Another important concept which widely used and very useful for studying of different problems in quantum information theory is coherent states or quasiclassical states which first introduced by Schrödinger in 1926 [27] and then it extended by Glauber [28] and Perelomov [29]. Coherent

states are applied for study of entangled nonorthogonal states, also they have vital importance in quantum optic [30, 31] and mathematical physics [29]. Bosonic entangled coherent state [32], SU(1,1), and SU(2) coherent states [33] are typical examples of entangled coherent states. Recently, much attention has been paid to continuous variable quantum information processing in which continuous variable type entangled pure states play a important role [34, 35, 36, 37]. For example, two-state entangled coherent states are used realize effective quantum computation [38] and quantum teleportation [39]. Two-mode squeezed vacuum states have been applied to quantum dense coding [40]. Therefore, it is an attractive subject to apply and study continuous variable type entangled pure states. One of these states is pair coherent state where preliminary concept of this state was presented by Agarwal [41, 42]. Agarwal suggested that the optical pair coherent state can be generated via the competition of four-wave mixing and two-photon absorption in a nonlinear medium. Another scheme has been suggested for generating vibrational pair coherent states via the motion of a trapped ion in a two-dimensional trap [43]. Since calculation of entanglement measures for high dimension state is difficult, it is a urgent task to find bound for entanglement measures [44, 45, 46, 47, 48, 49]. The basic aim of this paper is calculation of upper and lower bounds of D-concurrence over a family of non-Gaussian states, namely, the pair coherent state.

The organization of this paper is as follows: In sec. 2, we have review the pair coherent state, investigated their properties briefly and at the end, we indicate to Peres-Horodecki criterion. In sec. 3, we have calculated upper and lower bound of the D-concurrence over pair coherent state. The conclusion is given in sec. 4.

2 Pair coherent state: A state with non-Gaussian wave function

The pair coherent states (PCS) are regarded as an important type of correlated two-mode states, which possess prominent nonclassical properties [50] such as sub-Poissonian statistics, strong intermode correlation in the number fluctuations, squeezing of quadrature variances, and violations of Cauchy-Schwarz inequalities and they have been extensively studied for violation of Bell inequalities [51, 52]. Such states denoted by $|\zeta, q\rangle$ where are states of a two-mode radiation field [41, 42] with the following properties

$$\begin{aligned} ab|\zeta, q\rangle &= \zeta|\zeta, q\rangle \\ (a^\dagger a - b^\dagger b)|\zeta, q\rangle &= q|\zeta, q\rangle, \end{aligned} \tag{2-2}$$

where a and b are the annihilation operators associated with two modes, ζ is a complex number, and q is the degeneracy parameter. Pair coherent states can be explicitly expanded as a superposition of the two-mode Fock states, i.e.,

$$|\zeta, q\rangle = N_q \sum_{n=0}^{\infty} \frac{\zeta^n}{(n+q)!}|n+q, n\rangle, \tag{2-3}$$

where the normalization constant N_q is given by (I_q is the modified Bessel function of the first kind of order q)

$$N_q = [|\ \zeta\ |^{-q}\ I_q(2\ |\ \zeta\ |)]^{\frac{-1}{2}}. \tag{2-4}$$

The pair coherent state for q = 0 (corresponding to equal photon number in both the modes) is given by [53]

$$|\zeta, 0\rangle = N_0 \sum_{n=0}^{\infty} \frac{\zeta^n}{n!}|n, n\rangle, \tag{2-5}$$

subsequently $N_0 = \frac{1}{\sqrt{I_0(2|\zeta|)}}$ and $I_0(2\ |\ \zeta\ |)$ is the modified Bessel function of order zero. The coordinate space wave function is given by

$$\begin{gathered} \langle x_a, x_b|\zeta, 0\rangle = \\ N_0 \textstyle\sum_{n=0}^{\infty} \frac{\zeta^n}{n!}\langle x_a|n\rangle\langle x_b|n\rangle = N_0 \sum_{n=0}^{\infty} \frac{\zeta^n}{n!}\frac{1}{\sqrt{\pi}}\frac{H_n(x_a)H_n(x_b)}{2^n n!}\exp\left[-\frac{x_a^2+x_b^2}{2}\right], \end{gathered} \tag{2-6}$$

where $\langle x_a|n\rangle$ is a harmonic oscillator wave function given in terms of the Hermite polynomial as

$$\langle x_a|n\rangle = \frac{H_n(x_a)\exp(\frac{-x_a^2}{2})}{(2^n n!\sqrt{\pi})^{\frac{1}{2}}}. \tag{2-7}$$

It is clear from the Eq. (2-6) that the wave function of the pair coherent state is non-Gaussian. Now, we investigate the inseparability of the pair coherent states in light of Peres-Horodecki criterion, however the state of Eq. (2-5) has an obvious form of Schmidt decomposition. This reflects the fact that this state is an entangled state.

Perese-Horodecki criterion

Nonseparability for these states has been established using Peres-Horodecki criterion [16, 17, 54]. The Peres-Horodecki inseparability criterion is known to be necessary and sufficient for the (2×2) and (2×3) dimensional states, but to be only sufficient for any higher dimensional states. This criterion states that if the partial transpose of a bipartite density matrix has at least one negative eigenvalue, then the state becomes inseparable. The density matrix ρ corresponding to the state $|\zeta, 0\rangle$ (which is a infinite dimensional state) can be written as

$$\rho = \left(\sum_{n=0}^{\infty} C_{nn}|n,n\rangle\right)\left(\sum_{m=0}^{\infty} C^*_{mm}\langle m,m|\right), \tag{2-8}$$

where $C_{mm} = N_0\frac{\zeta^m}{m!}$. Partial transpose of Eq. (2-5) was shown to have negative eigenvalues and therefore the nonseparability

$$\lambda_{nn} = \frac{1}{I_0(2 \mid \zeta \mid)}\frac{\mid \zeta \mid^{2n}}{(n!)^2}, \forall n \tag{2-9}$$

$$\lambda^{\pm}_{nm} = \pm\frac{1}{I_0(2 \mid \zeta \mid)}\frac{\mid \zeta \mid^{n+m}}{(n!m!)}, \forall n \neq m. \tag{2-10}$$

One can in fact construct negativity $N(\rho)$ by finding absolute sum of negative eigenvalues in lieu of a computable measure of entanglement as

$$N(\rho) =\mid \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} \lambda_{nm}^{-} \mid=\mid 1 - \frac{e^{2|\zeta|}}{I_0(2 \mid \zeta \mid)} \mid, \forall n \neq m. \tag{2-11}$$

In the limit $\mid \zeta \mid\to 0$, $N(\rho) \to 0$ which is indicating that there is no entanglement, because in this limit only $|0, 0\rangle$ state will survive in Eq. (2-5) and all such states like $|n, n\rangle$ are separable.

3 D-concurrence

Recently, the novel measure named D-concurrence, has been proposed by Ma and Zhang, which in comparison with concurrence, has a advantages such as simplicity of its structure and accuracy of results [26]. For a mixed state, D-concurrence is defined by the convex roof, that is, defined as the average D-concurrence of the pure states of the decomposition, minimized over all decompositions of $\rho = \sum_i |\psi_i\rangle\langle\psi_i|$

$$D(\rho) = \inf \sum_i p_i D(\psi_i), \tag{3-12}$$

where p_i are real numbers which satisfy the following condition

$$\sum_i p_i = 1, \tag{3-13}$$

and ψ_i are pure states. Upper and lower bounds of D-concurrence defined by

$$[\det(I - \rho_A) - \det(I - \rho)] \leq D^2(\rho) \leq [\det(I - \rho_A)], \tag{3-14}$$

where $\rho_A = Tr_B\rho$ is the partial trace of ρ over subsystem B. It is too difficult to calculate the measure of entanglement for mixed states in high dimensions, therefore it is too vital to find the bounds for measures such as D- concurrence over states like pair coherent states. According to density matrix ρ in Eq. (2-8), we calculate upper and lower bounds of D-concurrence over pair coherent states in infinite dimensions.

Calculation of upper bound

First, we calculate upper bound i.e, $\det(I-\rho_A)$. According to Eq. (2-8), ρ_A is as following

$$\rho_A=\sum_n^{\infty}\mid C_{nn}\mid^2 |n\rangle\langle n|. \tag{3-15}$$

Therefore $I-\rho_A$ equals to

$$I-\rho_A=\sum_n^{\infty}(1-\mid C_{nn}\mid^2)|n\rangle\langle n|. \tag{3-16}$$

Eq. (3-16) is a diagonal matrix, so that its determinant is

$$\det(I-\rho_A)=\prod_n(1-\mid C_{nn}\mid^2), \tag{3-17}$$

this equation is upper bound of D-concurrence.

Calculation of lower bound

According to Eq. (3-14), to calculate the lower bound, $[\det(I-\rho_A)-\det(I-\rho)]$ should be calculate ($\det(I-\rho_A)$ has been calculated above). Elements of the matrix $A=I-\rho$ are as following

$$A_{ij}=\begin{cases}1-\mid C_{ii}\mid^2 & \text{if } i=j\\ -C_{ii}C^*_{jj} & \text{otherwise.}\end{cases}$$

Because $I-\rho$ is a matrix with infinite dimension, therefore it is too difficult to calculate its determinant. Here, to calculate $\det(I-\rho)$, first we should calculate determinant of matrixs $I-\rho$ with low dimensions such as 2×2, 3×3, 4×4... and then generalize it to high dimensions that process is as following

$$\det(I-\rho)_{(2\times2)}=1-\mid C_{00}\mid^2-\mid C_{11}\mid^2=1-\textstyle\sum_{n=0}^{1}\mid C_{nn}\mid^2$$

$$\det(I-\rho)_{(3\times3)}=1-\mid C_{00}\mid^2-\mid C_{11}\mid^2-\mid C_{22}\mid^2=1-\textstyle\sum_{n=0}^{2}\mid C_{nn}\mid^2$$

$$\det(I-\rho)_{(4\times4)}=1-\mid C_{00}\mid^2-\mid C_{11}\mid^2-\mid C_{22}\mid^2-\mid C_{33}\mid^2=1-\textstyle\sum_{n=0}^{3}\mid C_{nn}\mid^2$$

$$\vdots$$

$$\det(I-\rho)_{(N\times N)}=1-\mid C_{00}\mid^2-\mid C_{11}\mid^2-\mid C_{22}\mid^2-...-\mid C_{N-1,N-1}\mid^2=1-\textstyle\sum_{n=0}^{N-1}\mid C_{nn}\mid^2. \tag{3-18}$$

Therefore determinant of $(I-\rho)$ is

$$\det(I-\rho) = 1 - \sum_{n=0}^{N-1} \mid C_{nn} \mid^2, \tag{3-19}$$

where $N = 2, 3, ...\infty$. So, considering Eq. (3-14), the lower bound of D-concurrence is as following

$$\prod_n (1- \mid C_{nn} \mid^2) - 1 + \sum_{n=0}^{N-1} \mid C_{nn} \mid^2 . \tag{3-20}$$

At the end, lower and upper bounds of D-concurrence, for the pair coherent states, are

$$(\prod_n (1- \mid C_{nn} \mid^2) - 1 + \sum_{n=0}^{N-1} \mid C_{nn} \mid^2) \leq D^2(\rho) \leq (\prod_n (1- \mid C_{nn} \mid^2)) \tag{3-21}$$

where clearly

$$\sum_{n=0}^{N-1} \mid C_{nn} \mid^2 < 1. \tag{3-22}$$

4 Conclusion

In this paper we focus on the family of non-Gaussian states that are known as continuous variable or infinite dimensional system, and we have studied measure of D- concurrence on these states. Since the D-concurrence for high dimension mix state is difficult to calculate, it is a necessary to find bound for D-concurrence, hence we have computed upper and lower bounds of D-concurrence over the pair coherent states.

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